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THE DYNAMIC BEHAVIOR OF A DIGITAL ELECTROHYDRAULIC ACTUATOR

C. Brinckmann

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C. Brinckmann,
Institut für Flugführung,
Braunschweig

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16. Abstract An analysis of the dynamic behavior of the digital electrohydraulic actuator (DEHA) indicates that an appropriate locking mechanism may allow it to be used under dynamic and static loads. The increase in load capacity is accompanied by an increase in its irregularity of motion, which means increased dynamic loads on the elements attached to it. Cycle time is affected to a limited degree by loads, but inertial load capacity is seriously limited by the magnitude of permissible overshoot; this restriction rules out the use of the DEHA for operating aircraft control surfaces directly. Its use as an anticipatory control unit in a sequential control system is suggested. A locking mechanism helps reduce the danger presented by the inaccurate calibration of throttle orifices, which may become worse over prolonged periods of operation.

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Table of Contents

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1. Symbols

a	Initial acceleration
A	Area
A_T	Throttle orifice area
A_P	Piston area
A_R	Piston rod area
C_T	Coefficient for throttle orifice
f_C	Cycle frequency
F	Force
F_{stat}	Static positioning force
F_L	Constant load force
k_o	Relative overshoot
M	Mass of load
n	Number of pistons
P	Pressure
P_s	Supply pressure
P_{rev}	Pressure in reversed chambers
P_D	Pressure in differential piston chamber
P_0	Ambient pressure
ΔP	Pressure differential
P_R	Relief valve actuating pressure
P_F	Feed pressure
P_i	Internal pressure
Q	Throughput
t	Time
T	Piston travel time
T_L	Locking time
T_C	Cycle time
T_{Tr}	Piston travel time, unloaded, unmodified
T_{TrM}	Travel time with mass load
T_{TrL}	Travel time with constant load force
T^*	Fraction of travel time due to inertial load
T_{LV}	Operating time for locking valve
T_{PV}	Operating time for valves of individual pistons

Tr	Travel
Tr_a	Excess movement
Tr_b	Opposed movement
Tr_u	Usable travel
Tr_o	Overshoot travel
ΔTr	Quantization unit
v	Positioning velocity
ρ	Density of hydraulic fluid

<u>Subscripts</u>

max	Maximum value
out	Outflow, cut out
in	Inflow, cut in
res	For resultant motion
i	Running index $1 \leq i \leq n$

THE DYNAMIC BEHAVIOR OF A DIGITAL ELECTROHYDRAULIC ACTUATOR

C. Brinckmann,
Institut für Flugführung, Braunschweig

2. Introduction /9*

As part of the work on a "fly-by-wire" aircraft control system with digital electronic signal transmission, the concept of a control surface actuator which can process the digital input signals from the control system directly has been pursued during the last few years.

This is a digital electrohydraulic actuator (called DEHA in the following) for linear movement in the form of a series configuration of pistons which are hooked into or clasped by one another and are accommodated in a common housing [1, 3, 4] (Fig. 1).

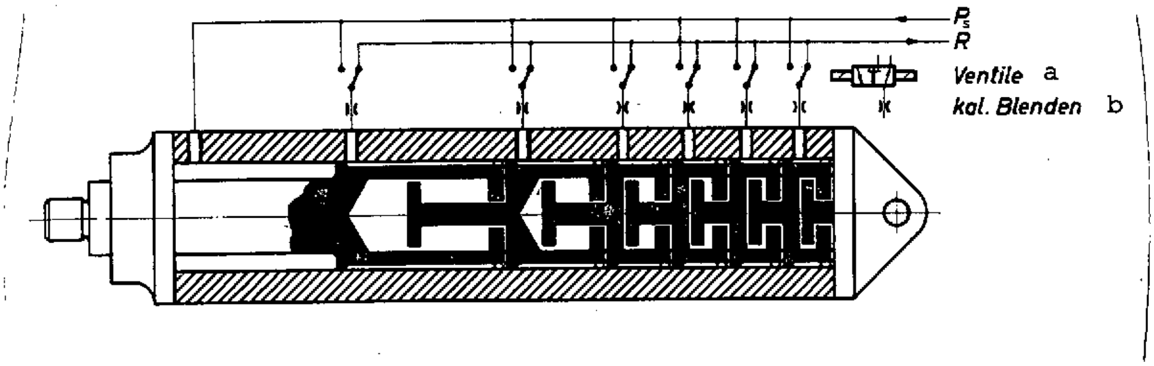


Fig. 1. Digital electrohydraulic actuator with six individual pistons, shown schematically.

Key: a. Valves
b. Calibrated orifices

An actuator of the DEHA type has the following advantages over analog servo actuators:

a) absolute positioning accuracy is determined only by the production precision with which the distances between piston stops can be maintained.

* Numbers in the margin indicate pagination in the foreign text.

b) relative resolution is a function of the number of pistons.

c) the rigidity of the actuator is a function only of dimensioning and choice of materials.

d) economy can be increased by reduced oil consumption.

e) the binary signal structure simplifies error recognition.

f) an electric digital-analog converter can be dispensed with in a digital control system.

g) the stability problems of a conventional analog actuator with servo amplifier, servo valve and the associated return system are not present, since the DEHA represents a pure open-loop system. /10

h) increased overall reliability should be achievable through the use of components which have demonstrated their reliability in similar forms (actuator cylinders) and through the utilization of newly developed components which provide the best conditions for their reliability through particularly simple modes of functioning (valves) [10, 11].

On the other hand, the following disadvantages exist:

a) component outlay and thus structural volume and weight/output ratio are increased.

b) dynamic behavior -- referring to transitional movements during position change -- is strongly affected by loads on the actuator.

This report is thus essentially meant to provide a contribution on the dynamic behavior of the DEHA.

3. Functional Description of the DEHA

To facilitate understanding, this functional description will be based on a model which consists of operating cylinders, arranged in series, with differential pistons (Fig. 2) which, like the DEHA pistons, are binary-weighted. Binary weighting means that the displacements of the individual pistons are in steps corresponding to the binary system, i.e. 1 (= 2^0), 2 (= 2^1), 4 (= 2^2), etc.

The signals delivered by the control system occur in parallel/ /binary form as the presence or absence of electric voltages.

Parallel means here that all binary variables can be called simultaneously. These signals are used to control magnetic valves which accomplish electrohydraulic transformation and digital-to-analog conversion of fluid flows (dynamic d-a conversion) by means of calibrated throttle orifices. The fluid flows in turn activate the individual pistons, which thus bring about hydromechanical conversion and d-a conversion into positional travel (static d-a conversion). The result -- in the form of positioning of the piston rod -- is obtained on addition of the /11
individual travels, which results from the series configuration of the piston.

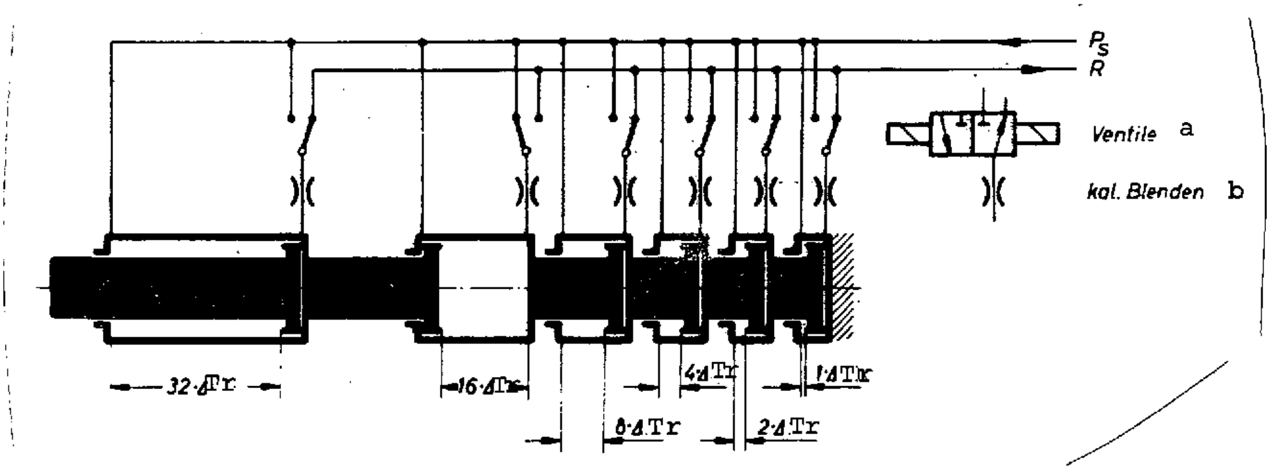


Fig. 2. Model of a digital actuator with six individual differential cylinders.

Key: a. Valves
b. Calibrated orifices

The piston to piston rod cross-sectional ratio for the individual differential pistons is taken to be

$$A_P : A_R = 2:1.$$

Since full supply pressure P_s always prevails in those piston chambers in which the piston rods reduce effective piston surface to one-half (Fig. 2), the forces exerted on the pistons from these sides are always $P_s \cdot A_P/2$. With the aid of the 3/2-way electric valves it is possible to connect the piston chambers opposite them with either the return or with the supply source, via throttle orifices. The forces generated in this manner counteract the above and are of magnitude zero (return pressure assumed to be ambient pressure) or $P_S \cdot A_P$. Depending upon the valve settings, the pistons are thus moved against their mechanical forward or return travel stops and are held there in each case with the same resultant static force

$$F_{stat} = P_S \cdot A_P - \frac{P_S \cdot A_P}{2} = \frac{P_S \cdot A_P}{2} . \qquad (1)$$

The number of possible positions (including 0) which result from corresponding combinations of the travels of individual pistons is 2^n for an actuator with n pistons (corresponding to n binary variables at the signal level). /12

If Tr_{max} is the sum of all piston travels and is thus maximum travel, then for the quantization unit ΔTr, i.e. for the smallest possible position difference, we obtain

$$\Delta Tr = \frac{Tr_{max}}{2^n - 1} \qquad (2)$$

With ΔTr we can also write the following for Tr_{max}:

$$s_{max} = \Delta Tr \cdot \sum_{i=1}^{n} 2^{i-1} = \Delta Tr \cdot (2^n - 1) \quad . \qquad (3)$$

The travel Tr_i of piston i, with a weighting of 2^{i-1}, is

$$Tr_i = \Delta Tr \cdot 2^{i-1}, \quad 1 \leq i \leq n, \qquad (4)$$

so the travel of the lowest-weighted piston (weighted 2^0) is equal to the quantization unit and thus also indicates the resolution of the actuator.

The model of a digital actuator shown in Fig. 2 requires a complete differential cylinder for each binary variable. The piston chambers continually connected with the supply source, which effect the return travel of the pistons, can be combined as one, however, so that all positioning pistons can be integrated into one housing in a simple manner. The pistons themselves must them all be fitted with seals and devices for limiting travel relative to one another. The DEHA shown in

Fig. 1 is developed in this manner, in which the limits of travel of the individual cylinders have been effected by means of T-shaped hooks.

4. Transition Behavior of the DEHA

4.1. Transition Behavior of the Unloaded DEHA

Due to the series configuration built into the DEHA, the motion of the piston rod represents the sum of the movements of individual pistons in the sense both of the instantaneous positions and of instantaneous velocities. The postion ultimately assumed is unequivocally defined by the piston stops. /13 Intermediate positions occurring during position changes are obtained from the time integral of piston velocities, however. The requirement of as steady a transition as possible from one position to another thus includes the requirement that travel speeds, like the travels, be binary-weighted and thus be proportional to them. This means the same travel time for all pistons, regardless of their direction of movement.

The travel velocity of piston i is determined by fluid throughput Q_i through the associated throttle orifice. For turbulent throughflow, the following orifice equation is valid to a good approximation:

$$Q_i = C_{Ti} \cdot A_{Ti} \cdot \sqrt{\frac{2 \cdot \Delta P_i}{\rho}}, \quad 1 \leq i \leq n, \tag{5}$$

where C_{Ti} = coefficient for throttle orifice i
A_{Ti} = area of throttle orifice i
ρ = density of hydraulic fluid
ΔP_i = pressure drop across throttle orifice i.

If valve switching time and piston mass are negligible, so that constant travel velocity can be assumed, then, with equation (5), we obtain the following for the travel time T_{Tri} for this piston:

$$T_{Tri} = \frac{Tr_i}{v_i} = \frac{Tr_i \cdot A_P \cdot \sqrt{\rho}}{C_{Ti} \cdot A_{Ti} \cdot \sqrt{2 \cdot \Delta P_i}} \tag{6}$$

where A_P = piston area
Tr_i = travel by piston i
v_i = velocity of piston i.

The pistons can be adjusted to equal travel times if the orifice cross sections are binary-weighted, making use of the throttle coefficients, and thus

$$Tr_i/(C_{Ti} \cdot A_{Ti}) = \text{const}$$

in equation (6). For a piston : piston rod cross-sectional ratio of 2:1 for the differential piston, a pressure P_{rev} of /14

$$P_{rev} = P_s/2$$

is set up in the chambers reversed for position change (if piston friction is neglected), due to the condition of force equilibrium, as long as the movement process has not ended. If it is assumed that return pressure is equal to ambient pressure, the pressure drop across the throttle orifices is then

$$\Delta P_i = P_s - P_{rev} = \frac{P_s}{2} ,$$

so that the same travel time $T_{Tri} = T_{Tr1} = T_{Tr}$ is obtained for all pistons with equation (6):

$$T_{Tr} = \frac{\Delta Tr}{C_{T1} \cdot A_{T1}} \cdot \frac{A_P \cdot \sqrt{\rho}}{\sqrt{P_s}} . \qquad (8)$$

For the unloaded DEHA, satisfactory transition behavior can thus be achieved with a constant positioning speed which is proportional to positioning travel.

4.2. Transition Behavior of the DEHA Under a Load

If a force F_L is applied to the piston rod of the DEHA from the outside, we obtain the following pressure drops ΔP_i across the throttle orifices from the condition of equilibrium of forces:

a) for pistons which are moving in the direction of the force,

$$\Delta P_{ia} = \frac{\frac{P_s \cdot A_P}{2} + F_L}{A_P} = \left(1 + \frac{F_L}{F_{stat}}\right) \cdot \frac{P_s}{2} \quad , \tag{9}$$

b) for pistons which are moving against the force,

$$\Delta P_{ib} = \frac{\frac{P_s \cdot A_P}{2} - F_L}{A_P} = \left(1 - \frac{F_L}{F_{stat}}\right) \cdot \frac{P_s}{2} \tag{10}$$

with F_{stat} from equation (1) and $1 \leq i \leq n$. As equation (6) shows, the travel times are /15

$$T_{Tri} \sim \frac{1}{\sqrt{\Delta P_i}} ,$$

so we can write the following for the ratio of travel times corresponding to the above pressure drops:

$$\frac{T_{Tria}}{T_{Tr}} = \frac{1}{\sqrt{1 + \frac{F_L}{F_{stat}}}} \tag{11}$$

$$\frac{T_{Trib}}{T_{Tr}} = \frac{1}{\sqrt{1 - \frac{F_L}{F_{stat}}}} \tag{12}$$

It can be seen from these two equations that the pistons traveling in the direction of the applied load force move faster and those traveling against the load move slower than at zero load. The result is that transition movement under load, with opposed piston movements, experiences a discontinuous course. For certain input signals, an initial movement in the wrong direction or an intermediate movement beyond the commanded position can even occur (e.g. in changing from position 3 to 4, i.e. from binary 011 to 100).

Two distinctions among cases must be made to describe these two effects, depending upon the direction of the force relative to the direction of the movement to be executed by the piston rod (resultant movement):

Case a)

The applied force has the same direction as the resultant. This means that the pistons traveling in the direction of the resultant have a shorter travel time, according to equations (11) and (12), than those moving in the opposite direction, so the piston rod can briefly go beyond the commanded position (overshoot). The absolute maximum overshoot occurs if all pistons participate in movement by only one quantization unit, i.e. between travel values $2^{n-1} - 1$ and 2^{n-1}. This is shown in Fig. 3 in the form of a travel/time diagram.

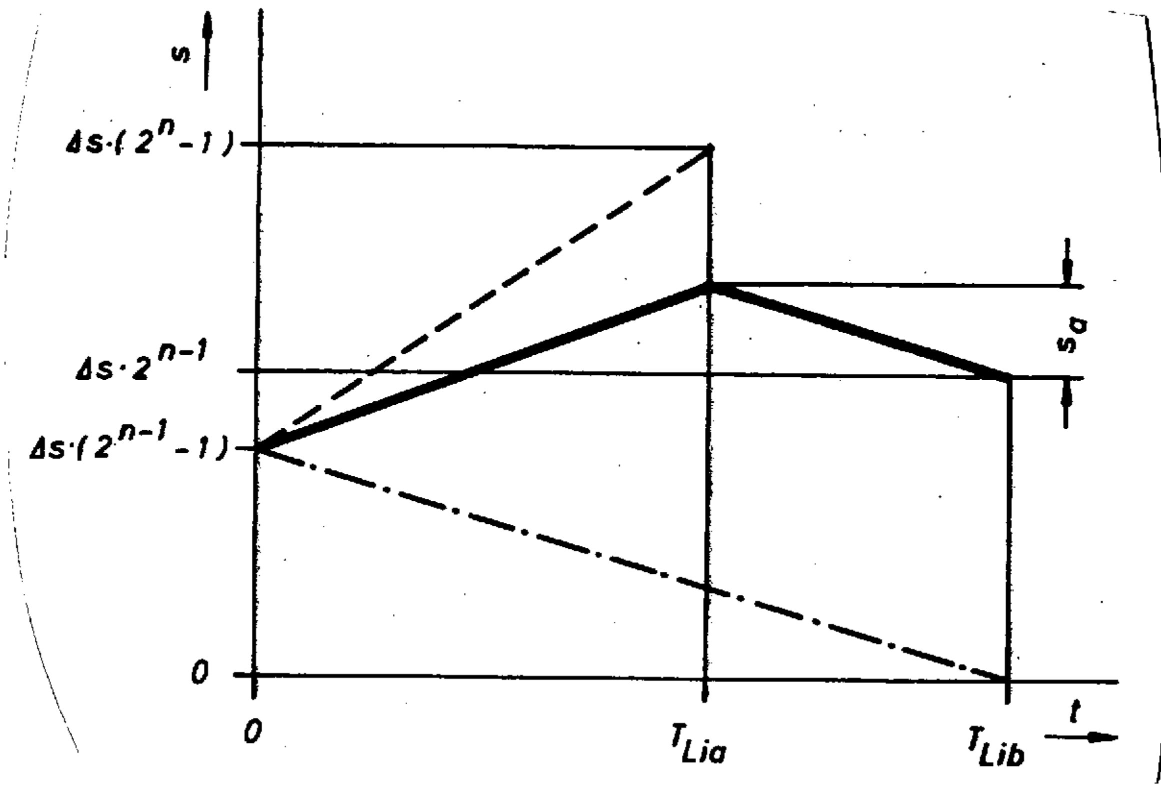


Fig. 3. Travel/time diagram of piston rod movement when loaded with a force in the direction of the resultant motion.

———— piston rod
– – – – pistons moving in the resultant direction
–·–·–· pistons moving against the resultant direction

$\Delta s = \Delta Tr$ $\qquad T_{Lia} = T_{Tria}$

$s_a = Tr_a$ $\qquad T_{Lib} = T_{Trib}$

From this diagram we can derive the relation /16

$$\frac{Tr_{a\,max}}{T_{Trib} - T_{Tria}} = \frac{\Delta Tr \cdot (2^{n-1} - 1)}{T_{Trib}}$$

(T_{Tria} and T_{Trib} from equations (11) and (12)) for maximum excess movement $Tr_{a\,max}$. By transforming this we obtain

$$\frac{Tr_{a\,max}}{\Delta_{Tr}} = (2^{n-1} - 1) \cdot (1 - \frac{T_{Tria}}{T_{Trib}}) \quad . \tag{13}$$

Upon substituting equation (2) we then obtain

$$\frac{Tr_{a\,max}}{Tr_{max}} = \frac{2^{n-1} - 1}{2^{n} - 1} \cdot \left(1 - \sqrt{\frac{1 - \frac{F_L}{F_{stat}}}{1 + \frac{F_L}{F_{stat}}}}\right) \quad . \tag{14}$$

This equation is shown graphically in Fig. 4 with piston number n as the parameter.

Case b)

The direction of the applied force is opposite to that of the resultant movement. This means that the pistons moving in the resultant direction have longer travel times, according to equations (11) and (12), than those moving in the opposite direction, so the piston rod can briefly execute a movement in the wrong direction (opposed movement). The absolute maximum opposed movement occurs, as in case a), between travel values $2^{n-1} - 1$ and $2^n - 1$. This is shown in Fig. 5 in the form of a travel/ time diagram. /18

From this diagram we derive the relation

$$\frac{Tr_{b\,max} + \Delta Tr}{T_{Trib} - T_{Tria}} = \frac{\Delta Tr \cdot 2^{n-1}}{T_{Trib}}$$

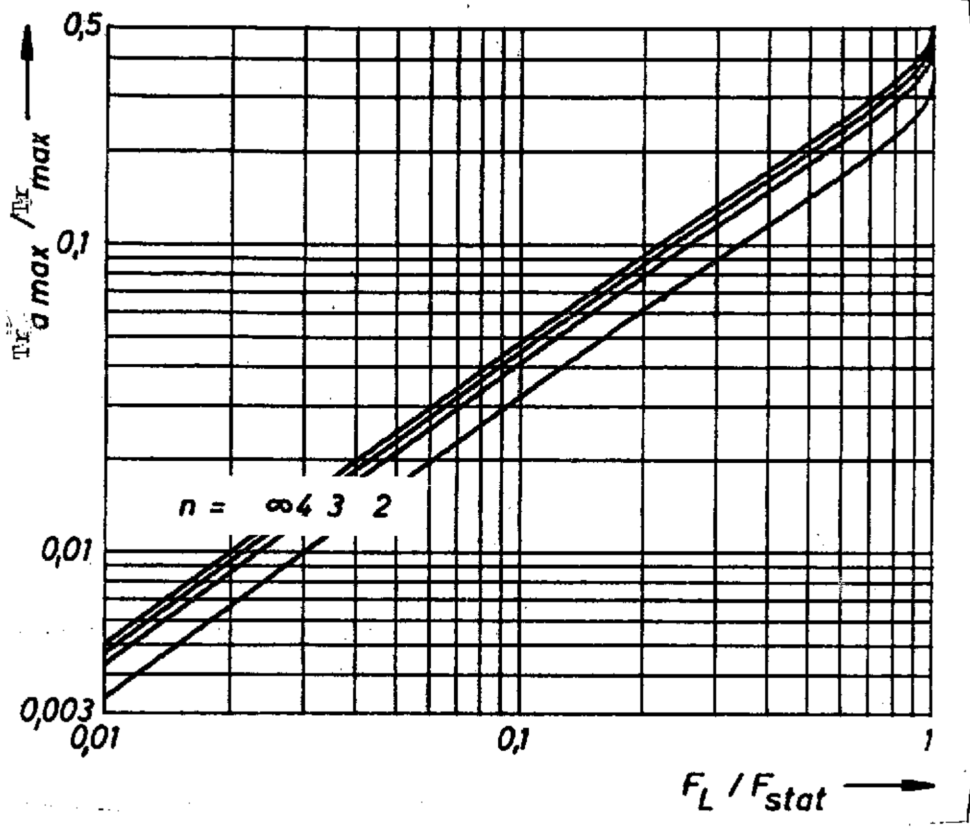


Fig. 4. Maximum relative excess movement as a function of relative load, with piston number n as the parameter.

[Note: Commas in numerals are equivalent to decimal points.]

(T_{Tria} and T_{Trib} from equations (11) and (12)) for opposed movement $Tr_{b\ max}$. By transforming this, we obtain

$$\frac{Tr_{b\ max}}{\Delta_{Tr}} = 2^{n-1} \cdot (1 - \frac{T_{Tria}}{T_{Trib}}) - 1 \quad . \tag{15}$$

Upon substituting equation (2) we then obtain /19

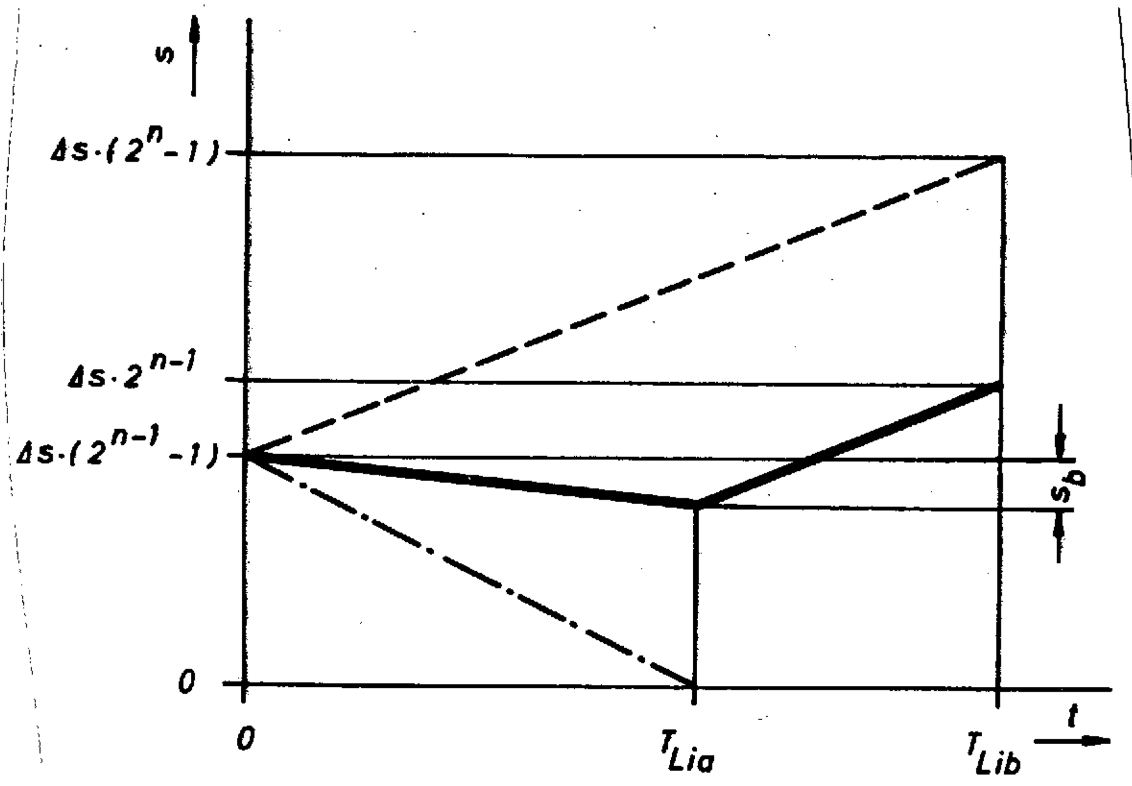


Fig. 5. Travel/time diagram of piston rod movement under loading with a force opposite to the resultant direction.

——— piston rod
------- pistons moving in the resultant direction
-·-·-·- pistons moving against the resultant direction

$s = Tr$
$s_a = Tr_a$
$T_{Lia} = T_{Tria}$
$T_{Lib} = T_{Trib}$

$$\frac{Tr_{b\ max}}{Tr_{max}} = \frac{2^{n-1}\cdot\left(1 - \sqrt{\dfrac{1 - \dfrac{F_L}{F_{stat}}}{1 + \dfrac{F_L}{F_{stat}}}}\right) - 1}{2^n - 1} \qquad (16)$$

This equation is defined only for $Tr_{b\ max}/Tr_{max} > 0$ (Fig. 5). It is shown graphically in Fig. 6 with piston number n as the parameter.

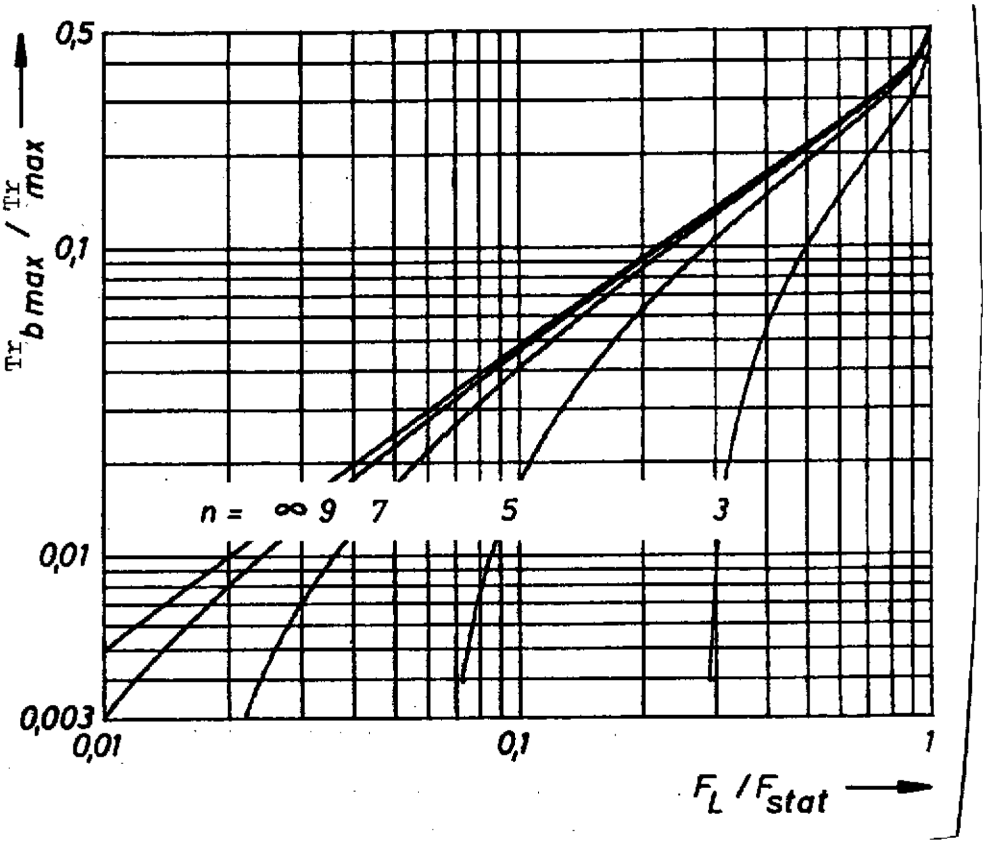


Fig. 6. Maximum relative opposed movement as a function of relative load, with piston number n as the parameter.

Figs. 4 and 6 make it clear that the errors in transitional behavior are a function of load. Impairment is so pronounced that an appreciable load capacity will be possible only in special applications, since inaccurate movements of the indicated magnitudes occur during motion by only one quantization interval. Thus the excess or opposing movement for a DEHA with nine pistons under a 20% load (F_L/F_{stat} = 0.2) is about 9% of the maximum value of 511 for positioning from a value of 255 to 256. This is about 45 travel units of motion error for a final position change of only one travel unit. /20

In the above considerations, it has always been assumed that the valve switching times are very much shorter than piston travel time T_{Tr}, so that they can be neglected for the movement process. If this condition is not satisfied, then care would have to be taken -- through suitable design of the valves -- that the individual throughputs are binary-weighted not only in thecompletely open state but also at every point in time during the opening process. This requirement certainly cannot be satisfied in practice with the necessary accuracy, so it is necessary to use particularly fast-acting valves.

5. Possibilities for Eliminating Dependence Upon Loads

5.1. Sequential Control, Compensation, Regulation, Limiting

So far, attempts have been made to deal with load-dependent transitional behavior through the application of a sequential control of high power amplification as shown in Fig. 7.

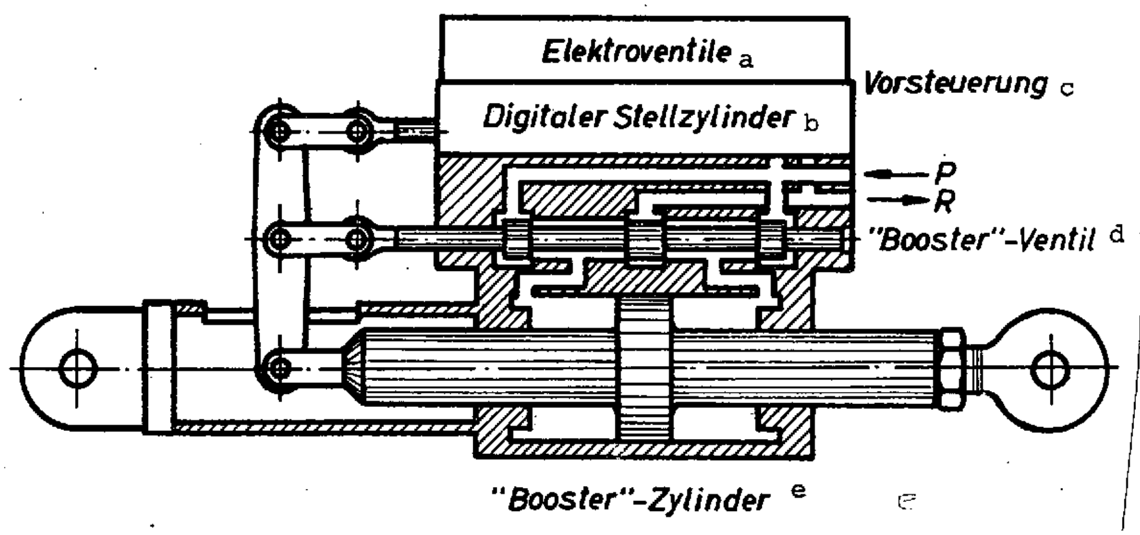


Fig. 7. Digital electrohydraulic actuator as an anticipatory unit for a booster cylinder.

Key:
- a. Electrical valves
- b. Digital actuator
- c. Anticipatory control
- d. Booster valve
- e. Booster cylinder

The DEHA then serves only to actuate a valve requiring very slight control forces, followed by a power (booster) cylinder. In spite of the small adjustment forces for normal operation, a power level of 200-300 daN is required, due to possible hydraulic locking of the valve slide, for the DEHA; this simultaneously establishes the smallest possible DEHA for a booster. /21

A realistic estimate of structural volume showed that up to an actuator output force of about 1000 daN, a DEHA by itself is smaller than the smallest possible DEHA with booster. A sequential control of this type is thus only reasonable if either no other possibility exists for eliminating load-dependent transition error or static load capacity is to be at least 1000 daN.

In order to make the DEHA also attractive below this load limit, several proposals were studied in detail with regard to their usability; it was kept in mind that the increase in reliability supposedly gained with the DEHA should not be lost again through additional, unreliable elements.

From this point of view, both the suggestion that the load be measured and compensated for by hydraulic control and the suggestion that flows through the individual valves be kept constant, regardless of load, by means of flow-regulating valves proved to be unacceptable. The attempt to achieve independence of loads by means of the phenomenon of flow limitation with cavitation in the throttle orifices failed because the efficiencies which could be achieved were too low.

5.2. The Locking Method

A solution to the problem lies, however, in the realization that transitional movements in the wrong direction and overshoots can only occur if opposing piston movements take place (overshoots due to inertial forces are not being considered here). A serial sorting of movements can be used to reduce this case to the errorless movement of a single piston, however. It is necessary here to block the piston associated with the highest changed bit in each case until the reversing movement of pistons at the subordinate levels have been completed. Expressed in graphic terms, the new stop for the directional output movement which takes place after the elimination of locking /22
is set during this time. For this method it is necessary and sufficient merely to lock the highest-level piston, simultaneously permitting the lowest outlay [4].

5.2.1. Direct Locking

Due to the high level of force, it does not appear to be desirable to produce mechanical locking of the piston rod with the (highest-level) output piston.

Hydraulic locking (with the aid of valves) which unequivocally locks the piston rod with the output piston in both directions of motion can only be achieved with a piston to which pressure can be applied from both sides (Fig. 8).

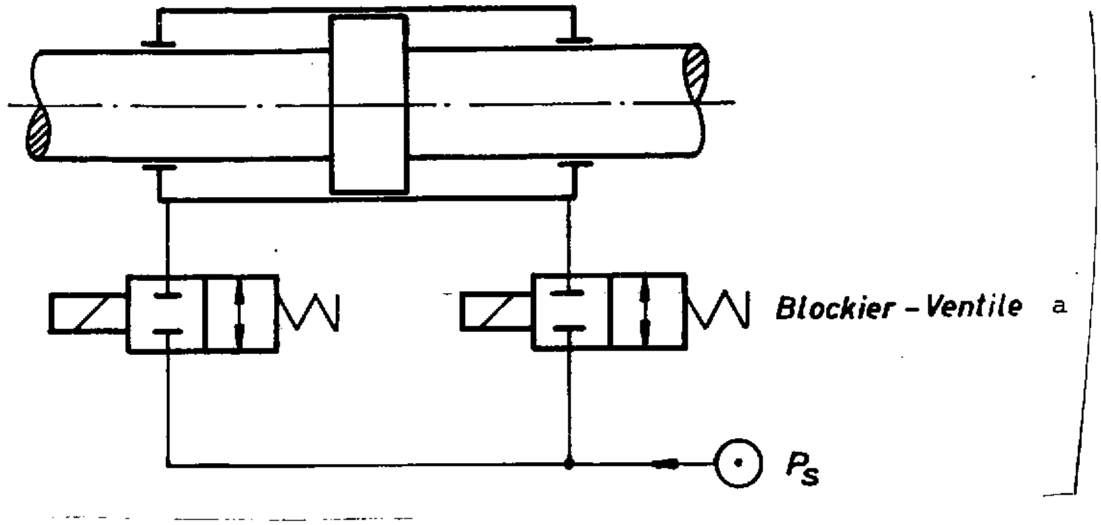


Fig. 8. Double-action hydraulic locking with valves (schematic).

Key: a. Locking valves

In addition to its task of effecting return travel, the piston chamber of the DEHA which is formed by the output piston and piston rod and in which full supply pressure P_S normally exists (differential piston chamber) can be employed for locking of the forward direction (cavitation would occur in the other direction). Any attempt, through design measures, to also lock the return direction through the addition of one more lockable chamber automatically produces a superfluous piston chamber, however, and thus undesirably lengthens the DEHA, which is already long anyway.

5.2.2. Locking with Compensating Circuits /23

The considerations discussed below were formulated for the reasons described above. A locking valve was to be used only for the differential piston chamber, and dynamic loadability was to be achieved with the aid of compensating circuits, without additional pistons.

The following absolute requirements are imposed under the assumption that the transition movement of the actuator must not necessarily begin exactly at the instant of unlocking of this locking valve, due to special dynamic requirements: if load capacity is supposed to be as great in dynamic terms as in static terms, i.e. for $A_P:A_R = 2:1$,

$$F_L = \pm F_{stat} = \pm \frac{P_s \cdot A_P}{2}$$

where $F_L > 0$ indicates tensile load and
$F_L < 0$ indicates compressive load,
then it is necessary to ensure, during the course of internal piston movements, that

a) no movements opposing the desired resultant movement occur (opposed movement), and

b) movements in the resultant direction can become no greater than the resultant (excess movement)

at the output piston, due to loading. We can differentiate here between two cases, depending upon the type of signal changes:

1) When a change is made in only 1 bit (e.g. from ...0101 to ...0001) and when changes are made in the same direction in more than 1 bit (e.g. from ...0000 to ...0111 or from ...1110 to ...0010), locking is not required since opposed and excess movements (detrimental movements) are not possible within the range of maximum compressive and tensile loads.

If a locking device is nevertheless actuated for these cases, it must be possible to draw additional fluid from the tank via a suction valve for negative resultants (i.e. for return travel) in order to prevent cavitation in the locking chamber (= differential piston chamber) under compressive load or prevent a vibrating movement under alternating loads. /24

2) When changes of opposite direction are made in the signal word (e.g. from ...0101 to ...1010), detrimental movements can be avoided if, in addition to the locking, the inflow and outflow throttle orifices are binary-weighted to correspond to the piston displacements. Since pressure in the locked chamber needs to be a maximum of $2P_s$ in order to avoid detrimental movements, a relief valve should avoid the occurrence of higher pressure amplitudes. Cavitation can again be prevented with a secondary-suction valve.

The effect on a DEHA modifed in this manner (Fig. 9) for the various signal and load states during the course of internal reversal processes, i.e. during locking, can be described as follows:

a) If the signal changes are designed to produce positive resultant movements (= advance travel) the resistances to inflow are lower than the resistances to outflow, due to the throttle orifices being weighted to correspond to piston displacements; for the pressure P_{rev} in the reversed chambers, we thus have $P_s/2 < P_{rev} < P_s$ in the case of locking. Since equilibrium can be maintained by just $P_{rev} = P_s/2$ under maximum compressive load (i.e. negative load direction), opposed movement is satisfactorily prevented. /25

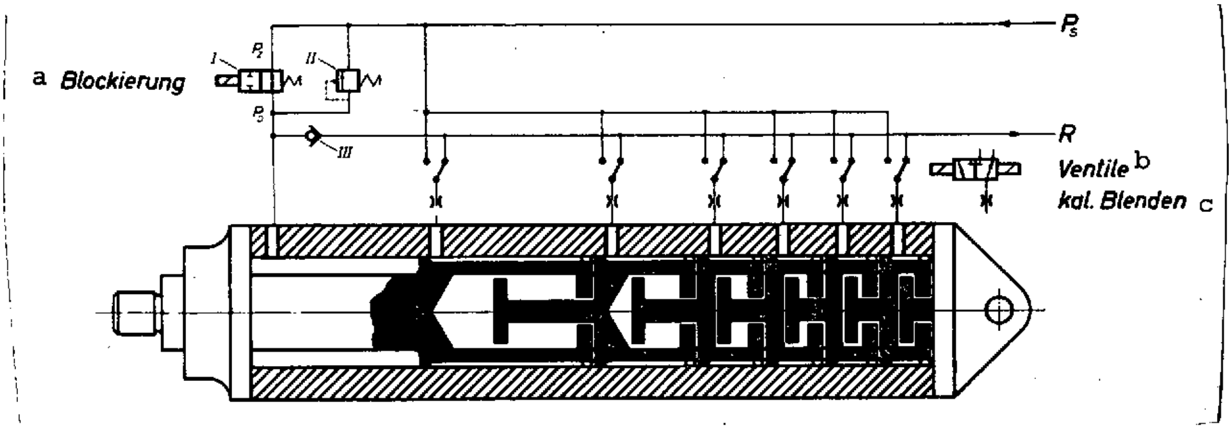


Fig. 9. Modified actuator of the type in Fig. 1, with locking valve I, relief valve II, secondary-suction valve III.

Key: a. Locking
b. Valves
c. Calibrated orifices

Without the relief valve II in Fig. 9, locking chamber pressure would rise to $3 \cdot P_s$ under maximum tensile load. With an overpressure valve set at only $2 \cdot P_s$, however, this case of loading just produces that state of motion which is equivalent to that of the unloaded DEHA without locking (with $P_{rev} = P_s/2$). Thus excess movements can likewise not occur then (assuming binary orifice steps).

b) When signal changes are made for negative resultant movements (= return travel), the resistances to inflow are greater than the resistances to outflow, due to the binary-stepped throttle orifices, and $0 \leq P_{rev} < P_s/2$ applies for pressure P_{rev} in the reversed chambers; under maximum tensile load, accordingly, maximum possible locking chamber pressure is $P_{D\,max} < 2 \cdot P_s$ (maximum tensile load produces, by definition, a pressure of P_s in the locking chamber, to which is added the pressure transformed from P_{rev} via the effective piston areas). Accordingly, opposed movement can be ruled out if the relief valve (II in Fig. 9) is set at (at least) $2 \cdot P_S$. Excess movements can likewise not occur (with binary-stepped throttle orifices) since the most unfavorable case of maximum compressive load just produces that state of movement which is equivalent to the unloaded DEHA without blockage (with $P_{rev} = P_S/2$). Cavitation and alternating direction of movement are prevented by the secondary-suction valve (III in Fig. 9) under a compressive or alternating load.

It should be noted, however, that detrimental movements are avoided only if

a) leakage losses at the piston seals remain negligibly small even under maximum loads; this requirement can be considered capable of being satisfied at the present state of the art in seal engineering;

b) elastic deformation of the seals and piston chambers (particularly that of the locking chamber) remains /26
sufficiently small under changing pressures;

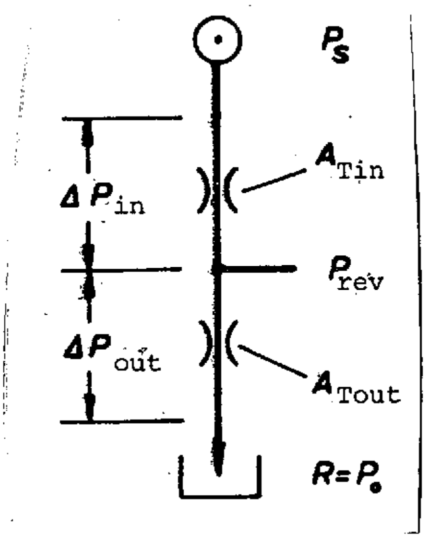
c) the valves operate at the same instant with similar time curves of pressure buildup, independently of the direction of operation. If the differences in operating time result in opposed movements which cannot be tolerated, an electronic logic unit should be used to ensure that in the case of positive resultants, the inflow valves are first actuated and then the outflow valves, slightly delayed, whereas in the case of negative resultants, the outflow valves and then the inflow valves are actuated.

5.2.3. Calculation of Required Cycle Time

Digital systems generally work in clocked cyclic operation, i.e. signal changes are transmitted only at certain time intervals. As compared with noncyclic operation, this means that the frequency band which can be transmitted has a cutoff. For the actuator, this offers the advantage of a lower frequency of valve actuation. If it is necessary that a commanded position be assumed prior to a new position change, a sufficiently long cycle time must be made available for a complete operating cycle.

Since the internal reversal processes and the resultant movement occur in succession when the locking method is applied, the required cycle time consists of the time for setting the locking, the maximum reversal time required, the time for releasing the locking, and the maximum time required for resultant movement.

Since positioning movements are avoided during the locking phase, a DEHA with a locking device can be considered as a chamber of constant volume during this time. Oil flows into it via the throttle orifices (connected in parallel) of the pistons which are cut in, and the same quantity of oil flows out via the throttle orifices (likewise connected with one another in parallel) of the pistons which are cut out. The pistons participating in internal movements can be viewed here as partitions which "float along." The schematic in Fig. 10 represents the hydraulic conditions.



C_T - All orifice values are the same magnitude

A_{Tin} - Sum of all orifice areas through which oil flows in

A_{Tout} - Sum of all orifice areas through which oil flows out

P_{rev} - Pressure in the chambers which are participating in reversal motion at this instant

ΔP_{in}, ΔP_{out} - Pressure drops across the equivalent orifices with A_{Tin} and A_{Tout}, respectively

Fig. 10. Hydraulic equivalent diagram for the processes occurring during the locking phase for the DEHA shown in Fig. 9.

From equation (5), we have the following for (turbulent) flow Q through an orifice:

$$Q \sim A_T \cdot \sqrt{\Delta P} \, .$$

Since $Q_{in} = Q_{out} = Q$ in our case, we can write the following (if the C_T values of all orifices are of the same magnitude):

$$Q \sim A_{Tin} \cdot \sqrt{\Delta P_{in}} = A_{Tout} \cdot \sqrt{\Delta P_{out}} \, . \tag{17}$$

From Fig. 10, $P_s = \Delta P_{in} + \Delta P_{out}$.

Then

$$\Delta P_{in} = \frac{P_s}{1 + \left(\frac{A_{Tin}}{A_{Tout}}\right)^2} \tag{18}$$

$$\Delta P_{out} = \Delta P_{in} \cdot \left(\frac{A_{Tin}}{A_{Tout}}\right)^2 \tag{19}$$

According to equation (6), travel time is

$$T_{Tri} \sim \frac{1}{\sqrt{\Delta P_i}} \, .$$

$T_{Tri} = T_{Tr}$ for $\Delta P_i = P_s/2$, so travel time T_x for other pressure drops $\Delta P_s \neq P_s/2$ is found to be

$$\frac{T_x}{T_{Tr}} = \sqrt{\frac{\frac{P_s}{2}}{\Delta P_x}} \, . \tag{20}$$

By substituting equations (18) and (19) into equation (20), we obtain the travel times for the pistons which are cut in and cut out:

$$\frac{T_{in}}{T_{Tr}} = \sqrt{\frac{1 + \left(\frac{A_{Tin}}{A_{Tout}}\right)^2}{2}} \tag{21}$$

$$\frac{T_{in}}{T_{Tr}} = \frac{1}{\frac{A_{Tin}}{A_{Tout}}} \cdot \frac{T_{in}}{T_{Tr}} \, . \tag{22}$$

The functions are shown graphically in Fig. 11.

We must distinguish between two cases in order to determine the time during which locking must be maintained so that those piston movements can be completed which would otherwise result in detrimental motion:

a) Positive Resultant Movements:

Detrimental movements could only be caused by the pistons moving in the negative directions (cut out). Positive resultants mean that $A_{Tin}/A_{Tout} > 1$. It can be seen from Fig. 11 that $T_{out} < T_{Tr}$ for this, so detrimental movements can no longer occur if we make locking time $T_L \geq T_{Tr}$.

b) Negative Resultant Movements:

In this case, $A_{Tin}/A_{Tout} < 1$. It can be seen in Fig. 11 that $T_{in} < T_{Tr}$ for this, so detrimental movement which could be produced by the pistons traveling in the positive direction (cut in) are likewise eliminated if we make locking time $T_L \geq T_{Tr}$.

Detrimental movements are thus avoided in each case if locking time T_L is made equal to the piston travel time T_{Tr} which is taken in the unlocked state if the pressure drop across the throttle orifices is $\Delta P = P_s/2$: /29

$$T_L = T_{Tr}. \tag{23}$$

If it is assumed that the coefficients C_T for all throttle orifices are of the same magnitude, then from equation (6), travel time without locking ($\Delta P = P_s/2$) is

$$T_{Tr} = \frac{Tr_i}{A_{Ti}} \cdot K, \qquad K = \frac{A_P \cdot \sqrt{\rho}}{C_{T_i} \cdot \sqrt{2 \cdot \Delta P_i}} \;. \tag{24}$$

Due to the "tuning" of orifices, this relation also applies to the general case in which Tr_i is the resultant Tr_{res} of the movements of various pistons and thus $A_{Ti} = A_{Tin} - A_{Tout}$:

$$T_{Tr} = \frac{|Tr_{res}|}{|A_{Tin} - A_{Tout}|} \cdot K \;. \tag{25}$$

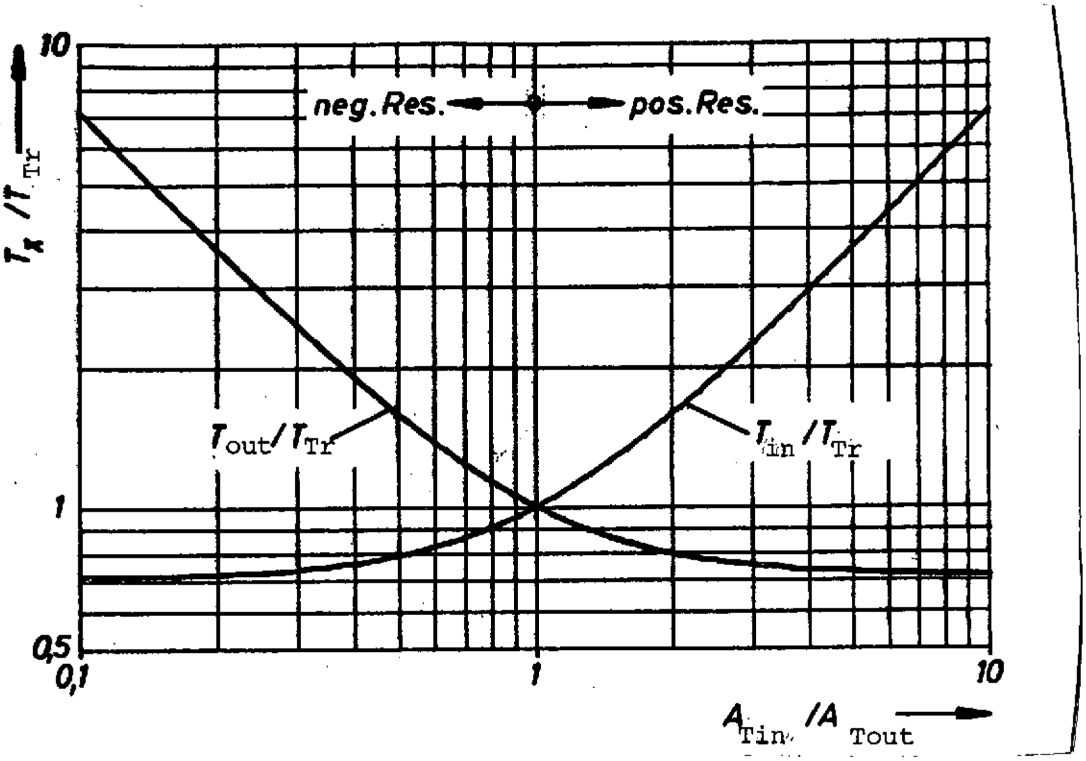


Fig. 11. Curves of travel times of the cut-in (T_{in}) and cut-out (T_{out}) pistons during the locking phase versus piston travel time (T_{Tr}) for the unlocked DEHA.

With locking, only the resultant movement can still occur after the end of the locking time, since all opposed movements have already been completed during this locking. All pistons traveling in the resultant direction participated in proportion to their weighting, since the same pressure drop ΔP_{in} or ΔP_{out} existed across all orifices for the cut-in or cut-out pistons, respectively, in each case. Thus all pistons switched in the resultant direction and the associated orifices participate in the resultant (= residual) movement after the release of locking. The travel time T_{res} which is necessary for this is a function of the absolute magnitude of the resultant Tr_{res} and the sum of the involved throttle orifice cross sections A_{Tres}. Since all movements opposing the resultant movement have already been eliminated during the locking phase, $A_{Tres} = A_{Tin}$ for positive resultant movements and $A_{Tres} = A_{Tout}$ for negative resultants. Thus

$$T_{res} = \frac{|Tr_{res}|}{|A_{Tres}|} \cdot K^*; \quad K^* = K, \text{ since } \Delta P_{res} = P_s/2. \quad (26)$$

By combining with equation (25) we obtain

$$\frac{T_{res}}{T_{Tr}} = \frac{A_{Tin} - A_{Tout}}{A_{Tin}} = \frac{\frac{A_{Tin}}{A_{Tout}} - 1}{\frac{A_{Tin}}{A_{Tout}}} \quad \text{for} \quad \frac{A_{Tin}}{A_{Tout}} > 1 \quad \text{(= positive resultants)} \tag{27}$$

and

$$\frac{T_{res}}{T_{Tr}} = \frac{|A_{Tin} - A_{Tout}|}{A_{Tout}} = \left|\frac{A_{Tin}}{A_{Tout}} - 1\right| \quad \text{for} \quad \frac{A_{Tin}}{A_{Tout}} < 1 \quad \text{(= negative resultants)} \tag{28}$$

These functions are shown graphically in Fig. 12. This mode of representation also makes it clear that it is sufficient to again make the same time available for the resultant movement as for the locking phase:

$$T_{res} = T_{Tr}. \tag{29}$$

The time T_{LV} required to set and release locking, like switching time T_{PV} of the valves for the individual pistons, is a function only of the type of valves used.

Fig. 13 provides a qualitative illustration of the changes in required time which result from using the locking mechanism if the DEHA otherwise remains unchanged. /31

If it must be ensured that commanded positions are also reached prior to a new position change, it can be seen from this representation that cycle frequency $F_C = 1/T_C$ must be decreased to less than half of the cycle frequency for the unmodified DEHA if the locking mechanism is used. Expressed otherwise, at a given cycle frequency, piston travel time T_{Tr} in the DEHA equipped with locking must be reduced to less than half. /32

Loads resulting from masses and forces leave the estimate of required locking time unchanged:

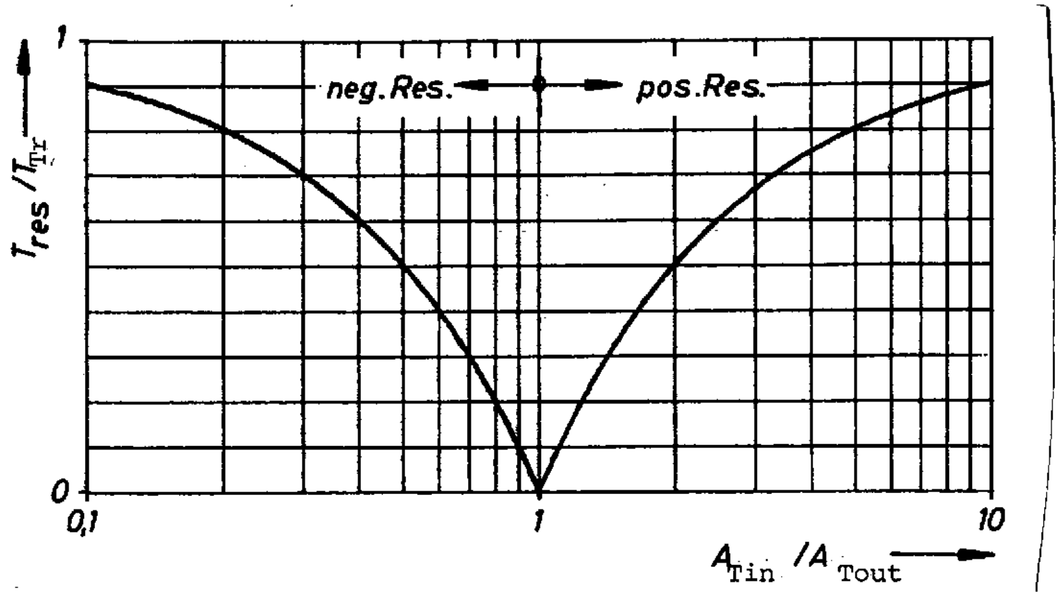


Fig. 12. Curves of resultant travel time (T_{res}) after release of locking over piston travel time (T_{Tr}) in the DEHA without locking.

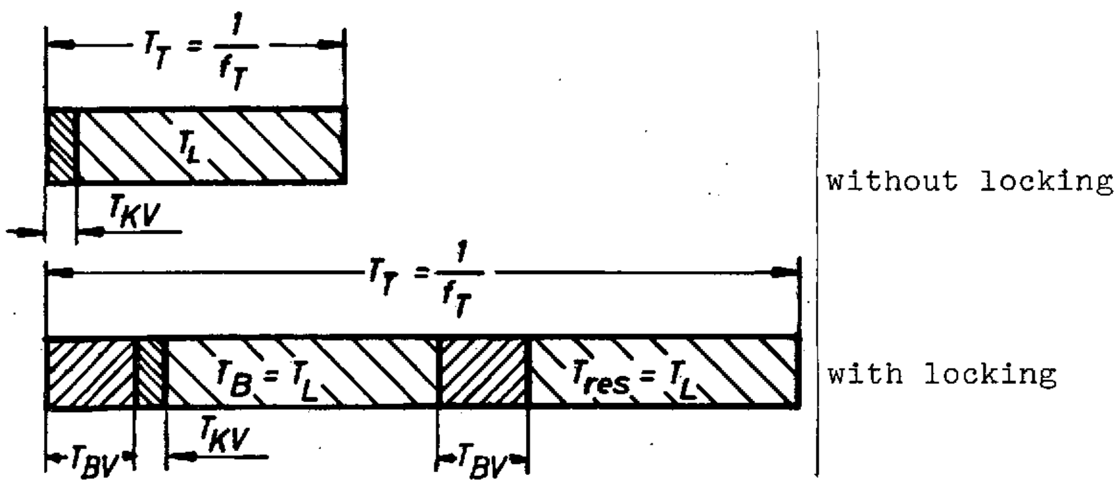


Fig. 13. Representation of time required for a position change in the DEHA with and without using the locking mechanism.

Key: $T_T = T_C$ [cycle time]
$f_T = f_C$ [cycle frequency]
$T_L = T_{Tr}$ [travel time]
$T_{KV} = T_{PV}$ [operating time for individual piston valves]
$T_B = T_L$ [locking time]; $T_{BV} = T_{LV}$ [operating time for locking valve]

$$T_L = T_{Tr}.$$

The time T_{res} required for the resultant movement is affected by this in the same direction as in the unmodified DEHA, however. Inertial loads call for a lengthening of T_{res}, whereas forces cause retardation or acceleration, depending upon effective direction.

In order to compute inertial effects upon the resultant movement, the DEHA can be reduced to a very simple system consisting only of a piston and cylinder.

In this simplified system (Appendix p. 41, Fig. A1), valid for the resultant movement, effective throttle orifice area is of the same size as for an unmodified DEHA only in the most unfavorable cases, according to the above derivations. In the majority of cases, it will be larger. Thus the throttle orifice areas of the unmodified DEHA must be used in calculating maximum possible resultant travel time.

The equation of motion for the simplified system is derived in the Appendix and reads

$$\ddot{Tr} + \frac{T_{Tr}^2}{Tr^2} \cdot a \cdot \dot{Tr}^2 = a$$

using T_{Tr} from equation (8) and

$$a = \frac{F_{stat}}{M} = \frac{P_s \cdot A_P}{2M} = \text{initial acceleration.}$$

This nonlinear differential equation is also solved in the Appendix. Its (implicit) solution for resultant travel time T_{TrM} with inertial load for travel Tr reads /33

$$T_{TrM} = T_{Tr} + \frac{Tr}{a \cdot T_{Tr}} \cdot \ln\left(1 + \sqrt{1 - e^{-2 \cdot \frac{a \cdot T_{Tr}^2}{Tr}}}\right) \tag{31}$$

or

$$\frac{T_{TrM}}{T_{Tr}} = 1 + \frac{T^*}{T_{Tr}} \tag{32}$$

where

$$\frac{T^*}{T_{Tr}} = \frac{Tr}{a' \cdot T_{Tr}^2} \cdot \ln\left(1 + \sqrt{1 - e^{-2 \cdot \frac{a' \cdot T_{Tr}^2}{Tr}}}\right) \tag{33}$$

For $Tr/(b \cdot T_{Tr}^2) < 1$, we can also write the following with an error of less than 5%:

$$\frac{T^*}{T_{Tr}} = \frac{Tr}{a \cdot T_{Tr}^2} \cdot \ln 2 = \frac{0{,}69 \cdot Tr}{a' \cdot T_{Tr}^2} \quad . \tag{33a}$$

Thus this formulation applies to $T^*/T_{Tr} < 0.69$. Equation (31) and the equation derived from it by transformation indicate resultant travel time for the most unfavorable case of motion from a standstill.

Equation (33) is shown graphically in Fig. 14. The way in which the nomogram is used can be easily seen from the example which has been drawn in (dashed line). For a piston travel time from equation (8) of T_{Tr} = 46 ms, a travel of Tr = 10 mm and a ratio of $a = F_{stat}/M = 10\ ms^{-2}$, the example indicates a lengthening of resultant travel time by $0.33 \cdot T_{Tr}$, to $1.33 \cdot T_{Tr}$, corresponding to T_{TrM} = 46 ms·1.33 = 61 ms, as the least favorable value.

Resultant travel time can likewise be determined with the nomogram in Fig. 14 for the case of loading with a mass and a constant force, e.g. resulting from friction. For this purpose we replace the parameter $a = F_{stat}/M$ with $a = (F_{stat} + F_L)/M$, in which F_L represents the constant load force. F_L is negative for forces opposing the motion (e.g. friction) and positive for forces in the direction of motion.

If the load consists only of a constant force F_L, then the equation given in the Appendix (A1) remains unaltered. From it we obtain /35

$$P_i = P_s - \frac{A_P^2 \cdot \rho}{C_T^2 \cdot A_T^2 \cdot 2} \cdot \dot{Tr}^2 \quad . \tag{34}$$

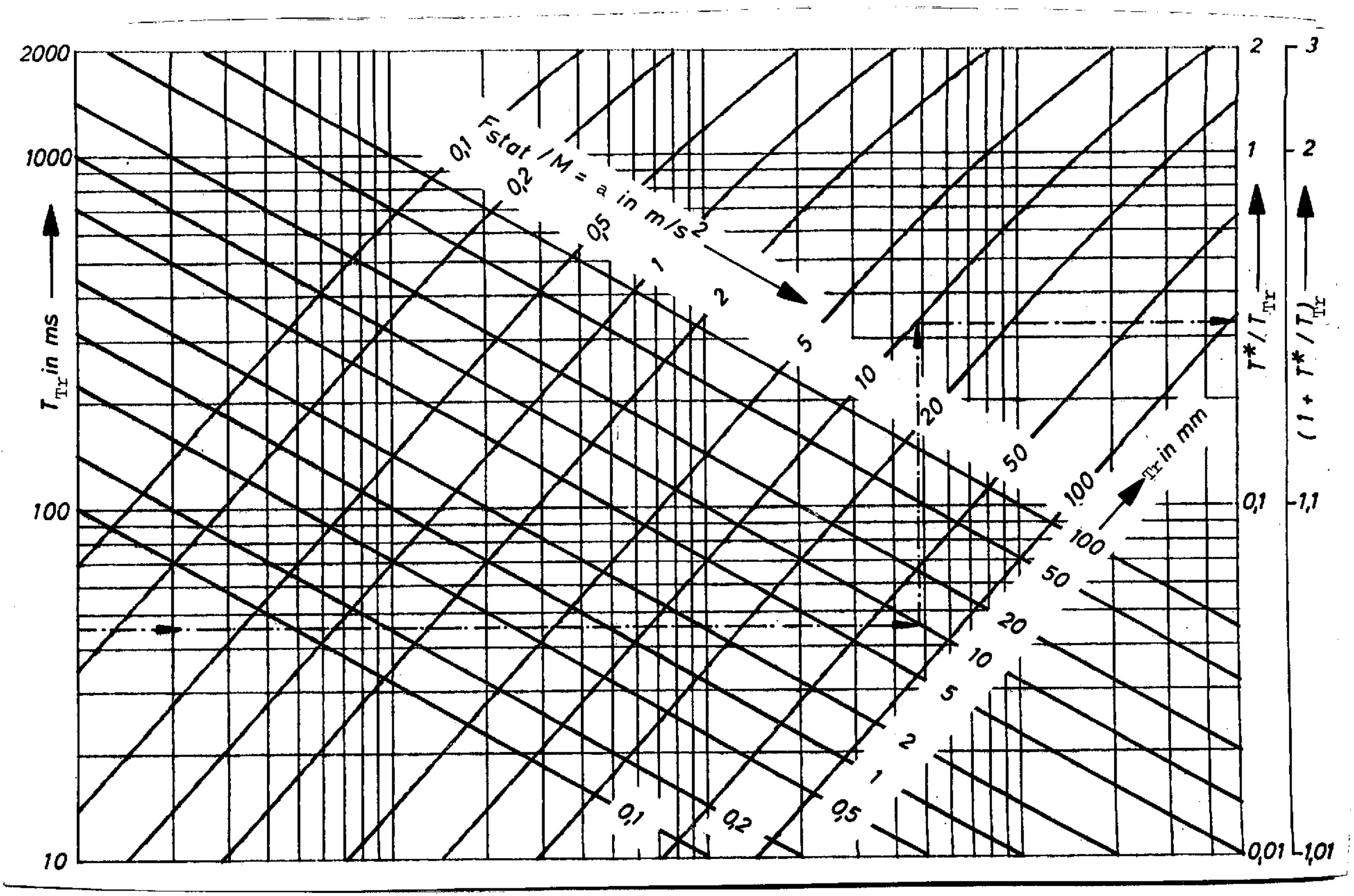


Fig. 14. Nomogram for determining maximum possible resultant travel time with an inertial load.

The right side of equation (A2) is changed, however:

$$P_i \cdot A_P - P_s \cdot \frac{A_P}{2} = - F_L \quad . \tag{35}$$

Equation (34) is substituted into equation (35) and, after introducing T_{Tr} from equation (8) and $F_{stat} = P_s \cdot A_P/2$, we obtain

$$\dot{T}r^2 = \left(\frac{Tr}{T_{Tr}}\right)^2 \cdot \left(1 + \frac{F_L}{F_{stat}}\right) \tag{36}$$

Now $\dot{T}r$ = const = Tr/t, and when the resultant movement is achieved, $t = T_{TrL}$, where T_{TrL} refers to the (maximum possible) resultant travel time under a load with a constant force.

Then, from equation (36),

$$\frac{Tr}{T_{TrL}} = \frac{Tr}{T_{Tr}} \cdot \sqrt{1 + \frac{F_L}{F_{stat}}}$$

and, after a slight transformation,

$$\frac{T_{TrL}}{T_{Tr}} = \frac{1}{\sqrt{1 + \frac{F_L}{F_{stat}}}} \tag{37}$$

Here F_L is again to be taken as negative for frictional forces and positive for forces in the direction of motion.

Equation (37) is shown graphically in Fig. 15 for simpler determination of resultant travel time, referred to the unloaded state.

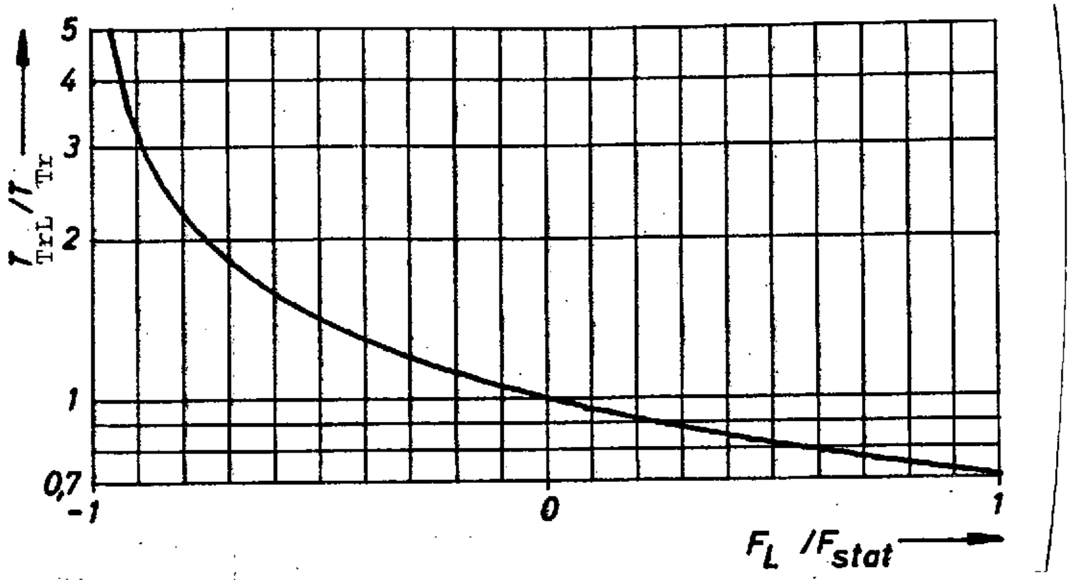


Fig. 15. Maximum possible relative resultant travel time under a load with constant force.

Thus the cycle time T_C which is required in the most extreme case in order to reach a position before a new position change can be initiated is as follows: /36

a) for an unmodified, unloaded DEHA:

$$T_C = T_{Tr} + T_{PV} \tag{38}$$

b) for a modified, unloaded DEHA:

$$T_C = 2\cdot(T_{LV} + T_{Tr}) + T_{PV} \tag{39}$$

c) for a modified DEHA loaded with a mass or mass and force:

$$T_C = 2\cdot(T_{LV} + T_{Tr}) + T_{PV} + T^* \tag{40}$$

or

$$T_C = 2\cdot(T_{LV} + T_{Tr}) + T_{PV} + \frac{0.69\cdot Tr}{a\cdot T_{Tr}} \quad \text{for } \frac{Tr}{a\cdot T_{Tr}^2} < 1 \tag{40a}$$

d) for a modified DEHA with a force load:

$$T_C = 2 \cdot (T_{LV} + T_{Tr}) + T_{PV} + T_{TrL} \quad (41).$$

6. Overshooting with an Inertial Load /37

6.1. Overshoot Damping

For the case of loading with masses, it is absolutely necessary to ensure that their kinetic energy is not conducted into the pistons in the form of excessive mechanical stresses during braking, particularly for input signals for minimum or maximum displacement. It has previously been proposed [4] that two additional passive pistons be added to the actuator, one of which is always connected with the tank and permits overshooting past maximum displacement, while the other is always under supply pressure and permits overshooting past minimum displacement. However, this provision generally enlarges the actuator by a multiple of the overshoot displacement which is achieved (Fig. 16).

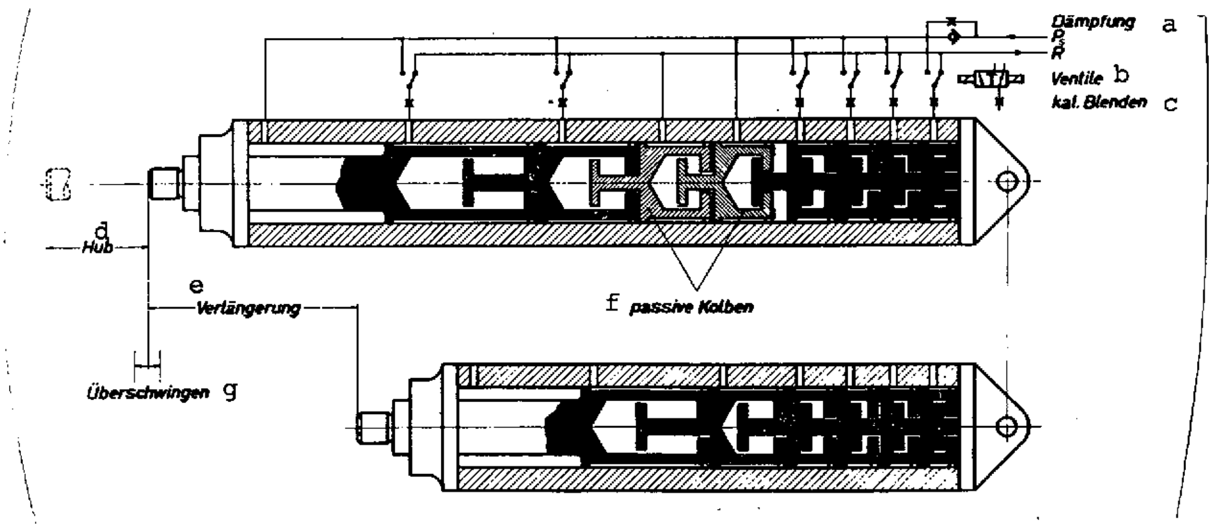


Fig. 16. Lengthening of the actuator caused by the insertion of passive pistons to dampen overshoot under inertial loads.

Key: a. Damping
b. Valves
c. Calibrated orifices
d. Travel
e. Lengthening
f. Passive pistons
g. Overshoot

A more economical method is to eliminate the occurrence of maximum and minimum displacement by limiting the repertory of possible input signals by an amount corresponding to the desired overshoot (e.g. ±10% of maximum travel). For a given static usable travel, this method requires only that the actuator be lengthened by the size of overshoot travel, but with a simultaneous increase in the quantization units.

For the actuator with passive pistons shown in Fig. 16, overshoot movements are damped with the parallel arrangement of a throttle orifice with a check valve in the supply line. The important dimensioning parameter for the throttle orifice is the pressure rise which occurs in the supply line on the actuator side during overshoot and which represents a special load on seals and materials. When turbulent flow passes through the orifice, this pressure rise and thus the damping force is proportional to the square of piston velocity. /38

The use of a pressure-limiting valve in place of the throttle orifice results in higher energy consumption for a given maximum pressure level, since the damping force constantly assumes its maximum value here, regardless of piston velocity.

Fig. 17 shows the actuator modified with locking, but also equipped for damping overshoot under an inertial load. Relief valve II, which has been set at a minimum of $P_R = 2 \cdot P_S$, in order to avoid detrimental movements in the transition phase of a position change, simultaneously serves as a damping valve for overshoot in the manner described above. The two check valves IV and V assume a rectifier function here, so only one relief valve is required for overshoot in both directions.

Since oil either from the differential piston chamber or from the first-level chambers can flow through the relief/damping valve only during the first half of each overshoot half-period in each case, a damping force likewise occurs only during this time (Fig. 18). /39

When the relief valve is set at $P_R = 2 \cdot P_S$, the overshoot movements are braked by a force which is twice as large as the static force, whereas return travel to the desired value is produced with the static force in each case.

In order to avoid having the damping force be dependent upon the desired position value, due to the calibrated orifices, for overshoot in the negative direction (i.e. when oil is forced out of the first-level piston chambers), check valves VII (Fig. 17) have been provided, with which the calibrated orifices can be circumvented in this operating situation.

The modification of the DEHA shown in Fig. 17 avoids detrimental movements under every type of load. A compilation

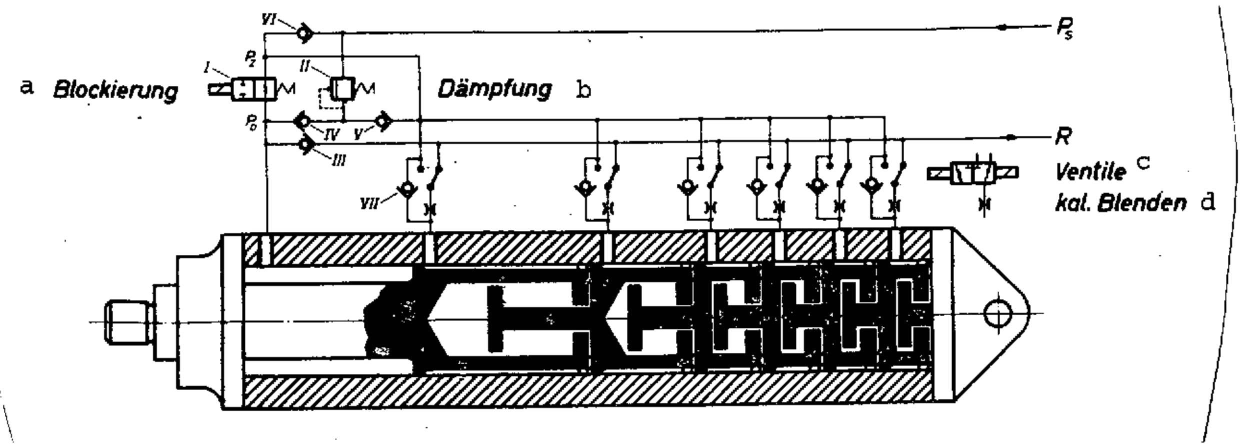


Fig. 17. Modified actuator of the type shown in Fig. 9, with additional provisions for damping overshoot with an inertial load.

Key: a. Locking
b. Damping
c. Valves
d. Calibrated orifices

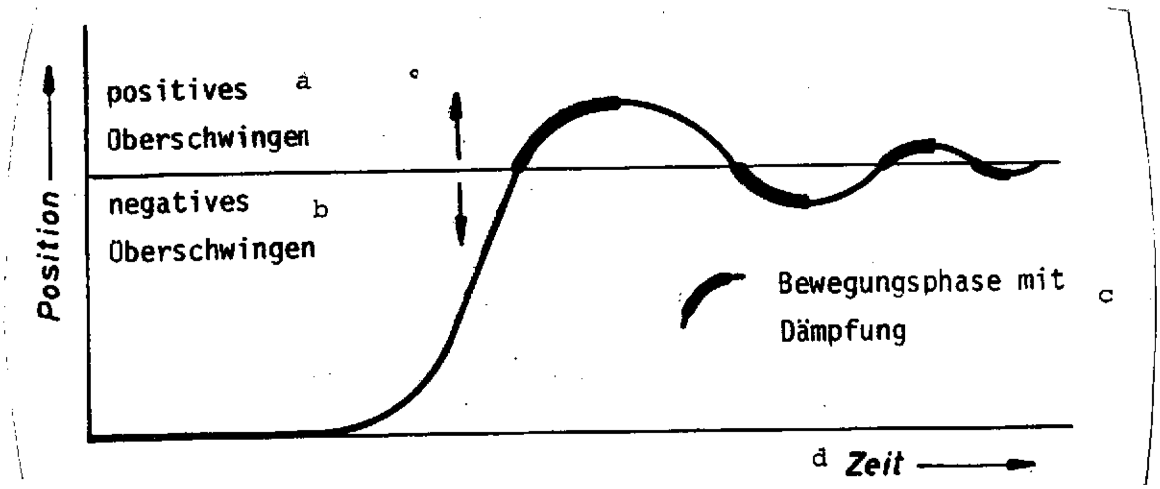


Fig. 18. Transient response of the inertially loaded actuator shown in Fig. 17.

Key: a. Positive overshoot
b. Negative overshoot
c. Phase of motion with damping
d. Time

of the activities of and loads on the elements of the locking mechanism is given in Table 1 for those conceivable operating states which are meaningful. To the extent that only certain operating states can occur in various applications, it is possible to determine the necessary minimum equipment for the locking mechanism with this table.

6.2. Calculation of Usable Travel /41

The calculations performed below apply to a modified actuator as shown in Fig. 17.

The calculation of usable travel requires a knowledge of overshoot travel. The maximum possible positioning velocity with which a position can be approached is decisive in determining maximum overshoot under an inertial load. From equation (A4), in the Appendix, after substituting the abbreviations from equation (A3a), we obtain the following for positioning velocity v after distance Tr has been covered:

$$v^2 = \frac{Tr^2}{T_{Tr}^2} \cdot \left(1 - e^{-\frac{2 \cdot T_{Tr}^2 \cdot a}{Tr}}\right) \tag{42}$$

where T_{Tr} and a are the same as in equation (30). The maximum possible travel Tr_{max} of the DEHA is determined by its design, through the number of pistons and the displacement of the lowest-level piston. Since the maximum overshoot travel Tr_{omax} must be permitted at the limits in each case, we find usable travel to be

$$Tr_u = Tr_{max} - 2 \cdot Tr_{omax}. \tag{43}$$

We define the proportionality factor for the maximum possible overshoot as

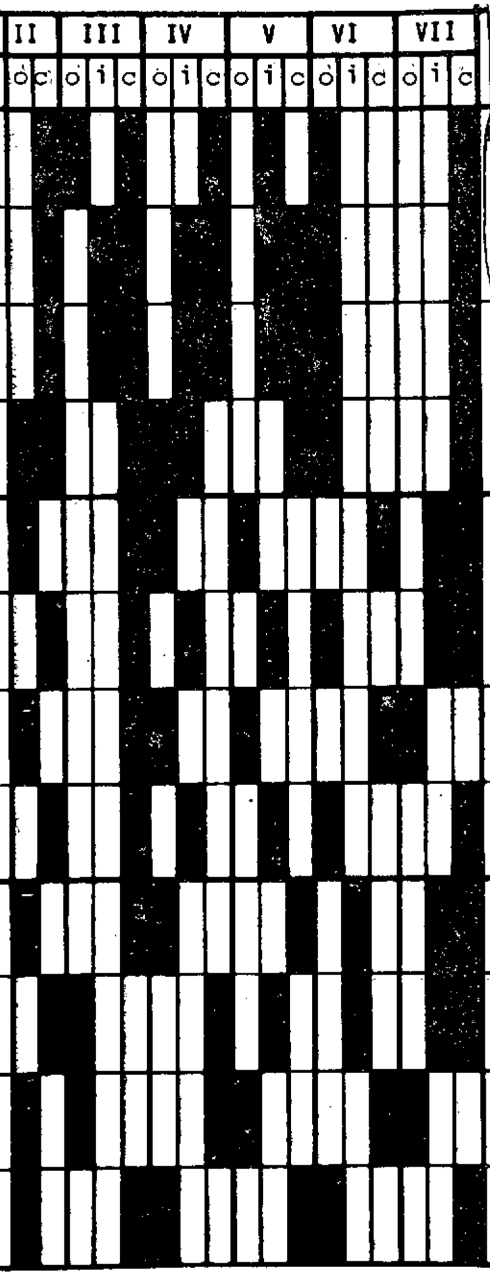
$$k_o = \frac{Tr_{omax}}{Tr_u}. \tag{44}$$

In the most unfavorable case, those calibrated orifices that participate in positioning movement from one of the limits of

TABLE 1. TABLE OF ACTIVITIES OF AND LOADS ON THE LOCKING MECHANISM

No	States	Valves I P_D	Valves I P_F	Remarks
1	I locked, neg. resultant, compressive load	0 P_s	P_s	$R_{infl.} > R_{outfl.}$
2	I locked, neg. resultant, tensile load	0 $2P_s$	P_s	$0 < P_{rev} < \frac{P_s}{2}$
3	I locked, pos. resultant, compressive load	0 $2P_s$	P_s	$R_{infl.} < R_{outfl.}$
4	I locked, pos. resultant, tensile load	P_s P_R	P_s	$\frac{P_s}{2} < P_{rev} < P_s$
5	I unlocked, pos. overshoot, forward travel	P_R	P_R	Cavitation in the zero-th levels
6	I unlocked, pos. overshoot, return travel	P_s	P_s	
7	I unlocked, neg. overshoot, return travel	P_R	P_R	
8	I unlocked, neg. overshoot, forward travel	P_s	P_s	Pressure in first levels < P_s
9	I locked, pos. overshoot, forward travel	P_R	P_s	Cavitation in 0-th levels. End of overshoot possible
10	I locked, pos. overshoot, return travel	0	P_s	
11	I locked, neg. overshoot, return travel	0	P_R	End of overshoot possible
12	I locked, neg. overshoot, forward travel	P_R	P_s	Pressure in first levels < P_s

Valves II (o c), III (o i c), IV (o i c), V (o i c), VI (o i c), VII (o i c)



P_D = pressure in differential piston chamber

P_F = pressure in valve feed line

P_R = relief valve actuating pressure: $2P_s < P_R < 3P_s$

P_s = supply pressure

P_{rev} = pressure in reversed piston chambers

P_T = tank pressure = 0

R = hydraulic resistance

o = open

i = inactive

c = closed

useful travel to the other would also participate in movement starting from the end positions. In order to determine maximum possible velocity at the limits of usable travel, we must therefore substitute the distance

$$Tr = Tr_u + Tr_{omax} = Tr_u \cdot (1 = k_o) \tag{45}$$

into equation (42).

The equation of motion for overshoot can again be derived with a simplified system (Fig. 19). /42

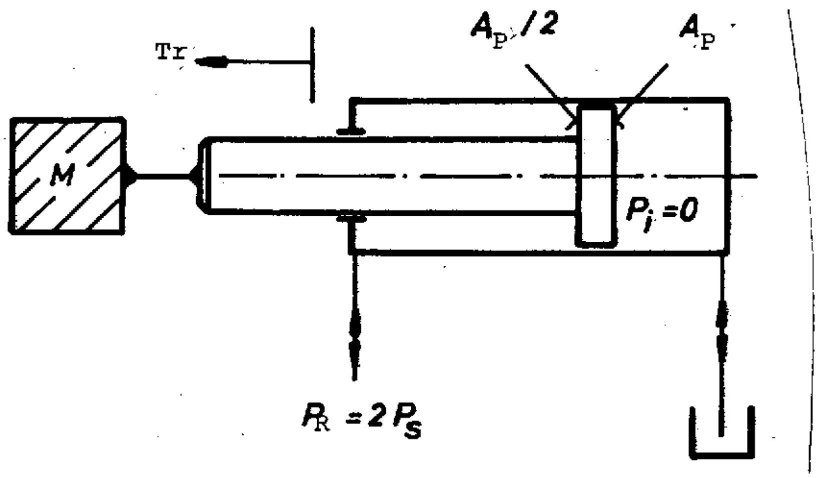


Fig. 19. Simplified system for calculating overshoot.

Due to relief valve II in Fig. 17, which has been set at $P_R = 2 \cdot P_s$, we find the forces on the piston to be

$$- 2 \cdot P_s \cdot \frac{A_P}{2} = M \cdot \ddot{Tr}$$

or

$$\ddot{Tr} = - 2 \cdot \frac{P_s \cdot A_P}{2 \cdot M} = - 2 \cdot a$$

where a is the same as in equation (30).

Thus the equation of motion reads

$$\frac{d^2Tr}{dt^2} = -2 \cdot a \ . \tag{46}$$

With the initial conditions at $t = 0$,

for distance, $\quad Tr = 0$
for velocity, $\quad \dot{T}r = dTr/dt = v$,

we can immediately write the solution

$$Tr = (v - a \cdot t) \cdot t, \tag{47}$$

and, from this

$$\dot{T}r = v - 2 \cdot a \cdot t. \tag{48}$$

Overshoot travel has reached its maximum value when $\dot{T}r = 0$. /43
The time $t = t_0$ required here is determined from this condition:

$$t_0 = \frac{v}{2 \cdot a} \ . \tag{49}$$

The end of overshoot travel $Tr = Tr_0$ is found from equation (47) by substituting the t_0 just determined:

$$Tr_0 = \frac{v^2}{4 \cdot a} \ . \tag{50}$$

The maximum value Tr_{0max} is apparently reached for $v = v_{max}$:

$$Tr_{omax} = \frac{Tr_u^{\,2} \cdot (1 + k_o)^2}{4 \cdot a \cdot T_{Tr}^2} \cdot \left(1 - e^{-\frac{2 \cdot T_{Tr}^2 \cdot a}{Tr_u \cdot (1 + k_o)}}\right)$$

Making use of equations (43) and (44), we obtain the following after a few transformations:

$$k_o = \frac{Tr_{max}}{4 \cdot T_{Tr}^2 \cdot a} \cdot \frac{(1 + k_o)^2}{(1 + 2 k_o)} \cdot \left(1 - e^{-\frac{2 \cdot T_{Tr}^2 \cdot a}{Tr_{max}} \cdot \frac{(1 + 2 k_o)}{(1 + k_o)}}\right)$$

For $k_o \leq 0.15$, it is possible to simplify this equation with an error $\leq \pm 1\%$ to obtain

$$k_o = \frac{Tr_{max}}{4 \cdot T_{Tr}^2 \cdot a} \quad \text{where} \quad a = \frac{F_{stat}}{M} \tag{51}$$

For conditions approximating reality, we thus have the relations between the important design parameters T_{Tr}, Tr_{max}, F_{stat}, load mass M and the maximum overshoot k_o referred to usable displacement.

Equation (51) is shown in Fig. 20 in the form of a nomogram. Its use can be seen from the example which is plotted in: For a piston travel time (from equation (8)) of T_{Tr} = 46 ms, a travel of Tr_{max} = 50 mm between end positions and a "ratio" of a = 100 ms^2, we obtain k_o = 0.059, corresponding to 5.9% overshoot (referred to usable displacement).

The useful travel which remains can be calculated by combining equations (43) and (44): /45

$$Tr_u = \frac{Tr_{max}}{1 + 2 \cdot k_o} \tag{52}$$

From equation (52) we obtain Tr_u = 50 mm/1.12 = 44.5 mm. Limiting travel to the usable displacement calculated in this manner is necessary for safety reasons, since it is possible to run through the entire usable travel in one cycle period when the

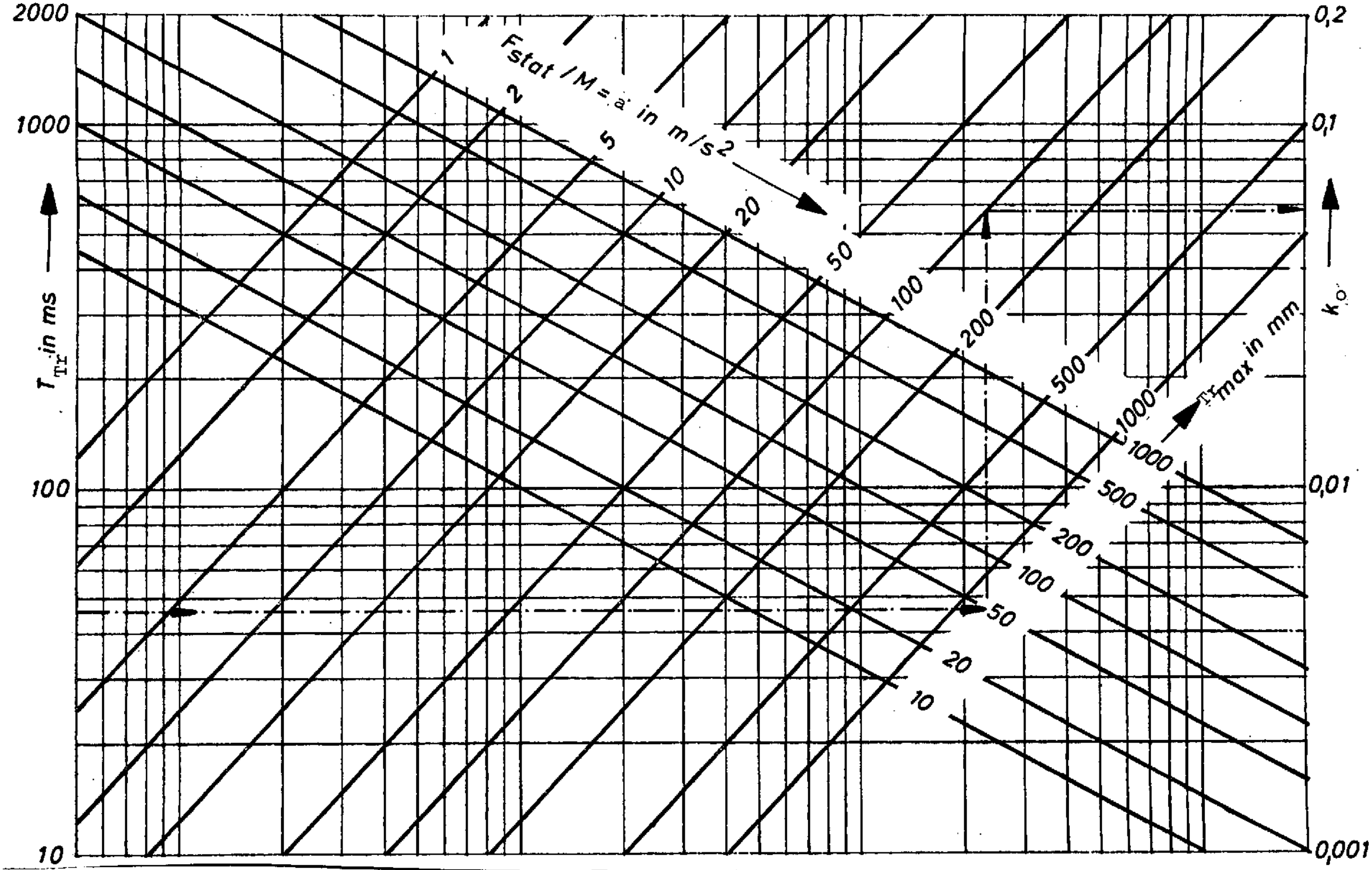


Fig. 20. Nomogram for determining maximum possible overshoot with an inertial load.

actuator is switched on. This must even be taken into consideration if, as the result of other measures, the mean positioning velocities and thus overshoot are very much lower in normal operation.

7. Summary

It has been demonstrated with the above analysis that the DEHA, which until now could only be used without loads, due to load-dependent shortcomings in transitional behavior, can be turned into an actuator which can be subjected to both dynamic and static loads, with the aid of the locking mechanism shown in Fig. 17. This advance should increase the attractiveness of the DEHA, in spite of the outlay required, which continues to be high.

The load capacity is obtained at the cost of an increased degree of irregularity of motion, since for a given cycle frequency, the travel velocity of the individual pistons must be more than doubled relative to an unmodified actuator, even for extremely short valve operating times. An increase in the degree of irregularity always means an increased dynamic load on the attached elements even if, as here, the magnitude of initial acceleration is not changed, since it is a function only of maximum cylinder positioning force.

The effect of the load-inducing forces and masses upon cycle time is slight (provided the forces do not reach the order of magnitude of static load capacity) and can be easily determined from the appropriate diagrams.

Inertial load capacity is quite considerably restricted by the magnitude of permissible overshoot. This restriction is so great that the direct actuation of control surfaces on aircraft /46
with a DEHA can be ruled out.

Thus even a modified DEHA can be considered for control actuators in aircraft only as an anticipatory control unit in a sequential control system with high force amplification as shown in Fig. 7.

A locking mechanism of an appropriately simple nature can still be very helpful for such an application, if the problem is to make the inaccurate calibration of throttle orifices harmless in its effect on initial motion of the actuator, since no experience has been accumulated regarding retention of the original calibration over relatively long operating times, and such retention must therefore be considered uncertain.

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Appendix: Effect of Inertial Loads on Travel Time /49

If hydraulic elasticity and damping are neglected, the equation of motion for the DEHA with a mass M acting as the load can be derived for its resultant movement with the aid of a very simplified system (Fig. A1).

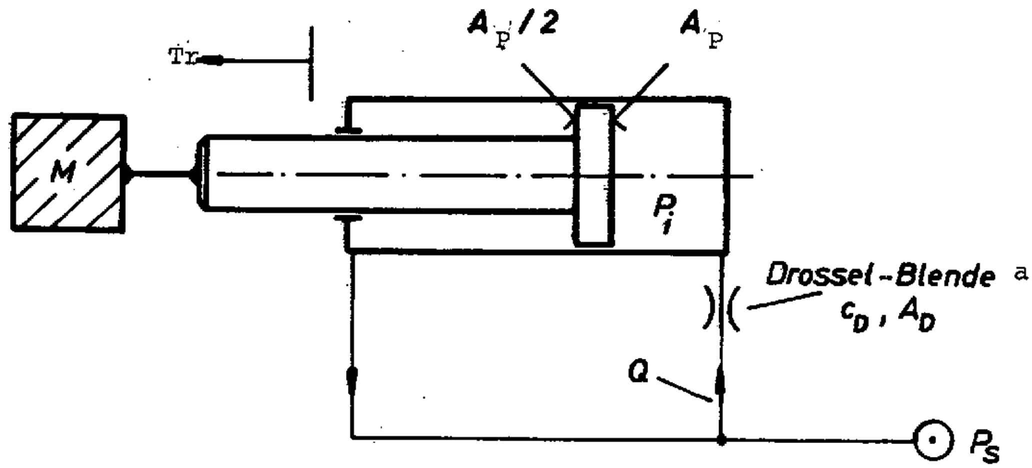

Fig. A1. Simplified system for calculating the effect of mass.

Key: a. Throttle orifice

The following apply to piston movement in only one direction:

a) the continuity condition:

$$Q = A_P \cdot \dot{Tr} = C_T \cdot A_T \cdot \sqrt{\frac{2 \cdot (P_S - P_i)}{\rho}} \qquad \text{(A1)}$$

and, from this,

$$P_i = P_S - \frac{A_P^2 \cdot \rho}{C_T^2 \cdot A_T^2 \cdot 2} \cdot \dot{Tr}^2 \; ;$$

b) equilibrium of forces:

$$P_i \cdot A_P - P_s \cdot \frac{A_P}{2} = M \cdot \ddot{Tr} \tag{A2}$$

From these two conditions we obtain the following equation of motion:

$$P_s \cdot A_P - \frac{A_P^3 \cdot \rho}{C_T^2 \cdot A_T^2 \cdot 2} \cdot \dot{Tr}^2 - P_s \cdot \frac{A_P}{2} = M \cdot \ddot{Tr}$$

$$\ddot{Tr} + \frac{A_P^3 \cdot \rho}{M \cdot C_T^2 \cdot A_T^2 \cdot 2} \cdot \dot{Tr}^2 = \frac{P_s \cdot A_P}{2 \cdot M} \tag{A3}$$ /50

In abbreviated notation:

$$\ddot{Tr} + b \cdot \dot{Tr}^2 = a \tag{A3a}$$

where

$$a = \frac{P_s \cdot A_P}{2 \cdot M} = \frac{F_{stat}}{M}$$ initial acceleration

and

$$b = \frac{A_P^3 \cdot \rho}{M \cdot C_T^2 \cdot A_T^2 \cdot 2} = a \cdot \frac{T_{Tr}^2}{Tr^2};$$ T_{Tr} from equation (8).

The equation of motion

$$\frac{d^2Tr}{dt^2} + b \cdot \left(\frac{dTr}{dt}\right)^2 = a$$

is a nonlinear second-order differential equation which, by substituting

$$\left(\frac{dTr}{dt}\right)^2 = z^2(Tr) = u(Tr)$$

and thus

$$\frac{d^2Tr}{dt^2} = \frac{dz}{dTr} \cdot \frac{dTr}{dt} = \frac{dz}{dTr} \cdot z(Tr) = \frac{1}{2} \cdot \frac{d(z^2)}{dTR} + \frac{1}{2} \cdot \frac{du}{dTr},$$

we can convert into a linear first-order differential equation:

$$\frac{du}{dTr} + 2 \cdot b \cdot u = 2 \cdot a \ .$$

From the homogeneous differential equation

$$\frac{du}{dTr} + 2 \cdot b \cdot u = 0,$$

making use of the familiary expression $u = e^{\lambda Tr}$ we obtain the following by substitution

$$\lambda \cdot e^{\lambda Tr} + 2 \cdot b \cdot e^{\lambda Tr} = 0$$
$$\lambda = -2b$$

and, as the solution, /51

$$u_h = C_1 \cdot e^{-2bTr}.$$

From the inhomogeneous differential equation

$$\frac{du}{dTr} + 2 \cdot b \cdot u = 2 \cdot a \ ,$$

making use of the method of variation of constants and the definition $u = C_1(Tr) \cdot e^{-2bTr}$, we obtain

$$\frac{du}{dTr} = \frac{dC_1}{dTr} \cdot e^{-2bTr} - 2 \cdot b \cdot C_1 \cdot e^{-2bTr}.$$

Upon substitution we obtain

$$\frac{dC_1}{dTr} \cdot e^{-2bTr} = 2 \cdot a$$

or

$$dC_1 = 2 \cdot a \cdot e^{2bTr} \cdot dTr.$$

Integration yields

$$C_1 = \frac{a}{b} \cdot e^{2bTr}$$

and, according to the defintion,

$$u_i = \frac{a}{b}$$

as the solution. Thus the general solution becomes:

$$u = u_h + u_i = \frac{a}{b} + C_1 \cdot e^{-2bTr} .$$

With initial conditions $t = 0$, $Tr = 0$, $dTr/dt = 0$, $z = 0$, $u = 0$, we obtain

$$C_1 = -\frac{a}{b},$$

and thus $u = a/b \cdot (1 - e^{-2bTr})$. Back-substitution of u yields

$$\left(\frac{dTr}{dt}\right)^2 = \frac{a}{b} \cdot (1 - e^{-2bTr}).$$

Integrating, we obtain /52

$$dt = \frac{dTr}{\sqrt{\frac{a}{b} \cdot (1 - e^{-2bTr})}}$$

and thus

$$t = \sqrt{\frac{b}{a}} \cdot \int \frac{dTr}{\sqrt{1 - e^{-2bTr}}} \quad .$$

We substitute

$$w = \sqrt{1 - e^{-2bTr}}$$

and obtain

$$dTr = \frac{w \cdot dw}{b \cdot (1 - w^2)} \quad .$$

Thus

$$t = \sqrt{\frac{b}{a}} \cdot \frac{1}{b} \int \frac{dw}{1 - w^2} = \frac{1}{\sqrt{a \cdot b}} \cdot \int \frac{dw}{1 - w^2} \quad .$$

Upon integration we obtain

$$t = \frac{1}{2 \cdot \sqrt{a \cdot b}} \cdot \ln \left| \frac{1 + w}{1 - w} \right| \quad .$$

Back-substitution of w yields

$$t = \frac{1}{2 \cdot \sqrt{a \cdot b}} \cdot \ln \left| \frac{1 + \sqrt{1 - e^{-2bTr}}}{1 - \sqrt{1 - e^{-2bTr}}} \right| \tag{A5}$$

After some transformations we have

$$t = \frac{1}{2 \cdot \sqrt{a \cdot b}} \cdot \ln \left| \frac{(1 + \sqrt{1 - e^{-2bTr}})^2}{e^{-2bTr}} \right|$$

$$t = \frac{1}{2 \cdot \sqrt{a \cdot b}} \cdot \left\{ 2bTr + 2 \cdot \ln \left(1 + \sqrt{1 - e^{-2bTr}}\right) \right\}$$

$$t = Tr \cdot \sqrt{\frac{b}{a}} + \frac{\ln \left(1 + \sqrt{1 - e^{-2bTr}}\right)}{\sqrt{a \cdot b}}$$

$$t = T_{TR} + \frac{Tr}{a \cdot T_{Tr}} \cdot \ln \left(1 + \sqrt{1 - e^{-2 \cdot a \cdot T_{Tr}^2/Tr}}\right) \qquad \text{(A5a)}$$ /52

with T_{Tr} from equation (8).

Now this is the (maximum possible) travel time of the DEHA with an inertial load if no opposing piston movements occur. It thus likewise applies to the resultant movement and will be designated T_{TrM}.

If that portion of the time caused by the mass is designated T^*, then, after normalization with T_{Tr}, we obtain

$$\frac{T_{TrM}}{T_{Tr}} = 1 + \frac{T^*}{T_{Tr}} \qquad \text{(A6)}$$

$$\frac{T^*}{T_{Tr}} = \frac{Tr}{a \cdot T_{Tr}^2} \cdot \ln \left(1 + \sqrt{1 - e^{-2 \cdot a \cdot T_{Tr}^2/Tr}}\right) \qquad \text{(A7)}$$

For $Tr/(a \cdot T_{Tr}^2) < 1$, this equation can be simplified to

$$\frac{T^*}{T_{Tr}} = \frac{Tr}{a \cdot T_{Tr}^2} \cdot \ln 2 = \frac{0.69 \cdot Tr}{a \cdot T_{Tr}^2} \qquad (A7a)$$

with an error of less than 5%, the restriction thus meaning that $T^*/T_{Tr} < 0.69$.

Equation (A7) has been shown graphically in Fig. 14 of the report as a nomogram; a practical example is also given there.